RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2019

SECOND YEAR [BATCH 2018-21]

Date : 21/12/2019 **MATHEMATICS [General]**

Time : 11 am – 2 pm **Paper : III** Full Marks : 75

(Use one Answer Book for Gr. A&C and another Answer Book for Gr. B)

Group – A

(*Answer any four questions*) [4×5]

1. Prove that if a plane has intercepts *l*, m, n on the axes and is at a distance 'p' from the origin, then $l^{-2}+m^{-2}+n^{-2}=p^{-2}$. [5]
2. Find the equation of the plane which bisects the obtuse angle between the planes $4x-3y+12z+13=0$ and $x+2y+2z=9$. [5]
3. Find the shortest distance between the lines
$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$$
$$\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}.$$
Find also the equations and the point of intersection in which it meets the lines. [5]
4. Find the equation of the sphere through the four points (0,0,0), (*a*,0,0), (0,b,0) and (0,0,c). [5]
5. Obtain the equation of the circle lying on the sphere $x^2+y^2+z^2-2x+2y-4z+3=0$ and having its centre at the point (2,2,-3). [5]
6. Find the equation of the right circular cone whose vertex is the origin, axis is the straight line $\frac{x}{2}=\frac{y}{-1}=\frac{z}{2}$ and semi-vertical angle is 60^{o}. [5]

Group – B

(*Answer any four questions*) [4×10]

7. a) Two linear simultaneous linearly independent equations with four unknowns are given below
$2x_1-x_2+3x_3+x_4=6$
$4x_1-2x_2-x_3+2x_4=10$
i) How many basic solutions are there?
ii) Find all of them.
iii) Discuss the nature of each and every basic solution. [2+3+2]

   b) Prove that the set $\{(x,y): x^2+y^2\le 4\}$ is a convex set. [3]

8. a) Find a basis for E^3 containing the vectors (1,1,2) and (3,5,2). [5]

   b) Solve the following L.P.P. by graphical method: [5]
Minimize $z=2x_1+7x_2$
subject to $5x_1+9x_2\le 45$
$2x_1+3x_2\ge 6$
$x_2\le 4$
$x_1, x_2\ge 0.$

9. a) A factory is engaged in manufacturing three products A, B and C which involve lathe work, grinding and assembling. The cutting, grinding and assembling times required for one unit of A are 2, 1 and 1 hours respectively. Similarly, they are 3,1,3 hours for one unit of B and 1,3,1 hours for one unit of C. The profits on A, B and C are Rs. 2 , Rs. 2 and Rs. 4 per unit respectively. Assume that there are available 300 hours of the lathe time, 300 hours of the grinder time and 240 hours of the assembly time. Formulate the problem as an L.P.P. in terms of maximising the profit. [5]

b) Solve the following L.P.P. by simplex method:

Minimize $z = 3x_1 - 2x_2$

subject to $x_1 - x_2 \le 1$

$3x_1 - 2x_2 \le 6$

and $x_1, x_2 \ge 0$. [5]

10. a) Use Charnes' M-method to solve the L.P.P.

Minimize $z = 4x_1 + 3x_2$

Subject to the constraints

$x_1 + 2x_2 \ge 8$

$3x_1 + 2x_2 \ge 12$

$x_1, x_2 \ge 0.$

[7]

b) Find the dual of the following primal problem:

Maximize $z = 2x_1 + 3x_2 + 4x_3$

subject to $x_1 - 5x_2 + 3x_3 = 7$

$2x_1 - 5x_2. \quad \le 3$

$3x_2 - x_3 \ge 5$

$x_1, x_2, x_3 \ge 0.$ [3]

11. a) Find the optimal solution of the following problem by solving it's dual: [6]

Minimize, $z = 10x_1 + 2x_2$

subject to $x_1 + 2x_2 + 2x_3 \ge 1$

$x_1 - 2x_3 \ge -1$

$x_1 - x_2 + 3x_3 \ge 3$

$x_1, x_2, x_3 \ge 0$

b) Prove that the dual of the dual is primal. [4]

12. a) Obtain the initial basic feasible solution of the following balanced transportation problem by VAM – method and calculate the corresponding cost: [5]

	D_1	D_2	D_3	D_4	a_i
O_1	16	21	11	15	30
O_2	7	14	9	29	40
O_3	19	17	23	33	53
b_j	22	35	25	41	

b) Find an initial basic feasible solution of the following transportation problem by Matrix-minima method: [5]

	D_1	D_2	D_3	D_4	D_5	a_i
O_1	2	11	10	3	7	4
O_2	1	4	7	2	1	8
O_3	3	9	4	8	12	9
b_j	3	3	4	5	6	

13. a) Find the optimal assignment cost from the following cost matrix. [6]

	X	Y	Z	U	V
I	3	5	10	15	8
II	4	7	15	18	8
III	8	12	20	20	12
IV	5	5	8	10	6
V	10	10	15	25	10

b) Write down the standard form of the following L.P.P :

Minimize $z = 5x_1 - 8x_2$

subject to $x_1 + x_2 \le 4$

$2x_1 + 3x_2 \le 6,\ x_1 \ge 0, x_2$ is unrestricted in sign. [4]

Group – C

(*Answer <u>any three</u> questions*) [3×5]

14. Compute the relative error and the relative percentage error in computing $f(x) = x^3 + 3x^2 - x$ for $x = \sqrt{2}$ (Take $\sqrt{x} = 1.414$ and increment in x = 0.00005). [3+2]

15. Construct the difference table and compute f (0.5) by Newton's Forward interpolation formula from the following table: [5]

x	0	1	2	3
f(x)	1	2	11	34

16. Using appropriate interpolation formula find the value of f(0.37) from the following data: [5]

x	0.00	0.10	0.20	0.30	0.40
f(x)	1.0000	1.2214	1.4918	1.8221	2.2255

17. Evaluate $\int_0^5 \frac{dx}{1+x}$, correct upto 4-significant figures, taking five equal sub-intervals by Trapezoidal rule. [5]

18. Use Newton-Raphson method to evaluate a positive root of the equation $x^3 - 5x + 3 = 0$, correct to three significant figures. [5]

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